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T1 ________________	**47577**	F1 ________________
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T4 ________________	Problem Chosen **B**	F4 ________________

2016
MCM/ICM
Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)

Type a summary of your results on this page. Do not include
the name of your school, advisor, or team members on this page.

In this paper, we present a model for determining the economic attractiveness of space debris removal for a private company. Given any combination of systems and parameters, our model can predict the company's maximum profit and their ideal combination of debris removal systems, enabling them to decide if it is economically attractive. Our model is very robust, and takes into account a wide range of costs, benefits, and risks, but it can still be solved using software the company already owns – even Microsoft Excel!

There are many space debris removal systems currently in development, and we cannot guarantee which systems would be available to a given company. To handle this, we develop a generic model for a debris removal system that can fit any option presented to the company. This model includes cost and risk parameters for various aspects of the system, as well as how often each system can be deployed. The parameters of this model make it easy for a company to explore various "What if?" scenarios.

Once a company has modeled all of its potential options, our model incorporates all of them into a detailed linear program. This program intelligently reports the optimal combination of systems that will maximize their profit, while taking multiple factors into account. It also provides other relevant information, such as revenue vs. cost breakdowns, the expected number of deployments per year, and detailed risk analysis.

In this paper we also provide a detailed analysis of several current debris removal methods. Our model was able to intelligently select the most profitable option for our theoretical company, no matter how many years the program ran or what the company's requirements were.

Also, since economic attractiveness means different things for different companies, we created a sub-model that provides an innovative approach to avoiding collisions with space debris. This sub-model determines the paths of debris so that collisions can be prevented, even with only the most basic information.

So, You Made a Mess In Space?

February 1, 2016

Contents

1 Introduction to Problem

The problem we chose to solve was to find an economically attractive way for a private company to reduce the amount of space debris orbiting the Earth. In the case that no affordable opportunity presented itself, we then had to provide innovative methods of avoiding collisions with space debris.

We developed a model that can determine the optimal combination of debris removal systems to maximize the profits of a private company while also meeting various requirements. We performed a qualitative analysis of many different debris removal methods and narrowed the search for the optimal method down to a few promising combinations. We proceeded to perform quantitative analysis on these methods as an example of what the companies would look at and how they can make their decision.

In case a private company using our model should decide that the optimal solution provided is not affordable for them, we also provide a detailed method for predicting the likely positions of space debris and where possible collisions may occur. We performed a runtime analysis of this sub-model and made recommendations of possible improvements that the company may desire to implement.

2 Assumptions and Justifications

Our model makes the following assumptions:

1. Since most space debris is found in low-Earth orbit (LEO), we can focus our efforts there and not on other altitudes.[17]
2. International politics will not interfere with our efforts. As justification, we present this quote from former NASA Chief Scientist for Orbital Debris: "Once an effective debris removal capability is developed, operations are likely to be financed and conducted under an international agreement."[1]
3. For simplicity, we assume all debris removal systems can be built and deployed independently of each other. This allows us to study combinations of alternatives as well as individual methods.
4. We will only focus on large pieces of debris. Our sources indicated that targeting large space debris would have the greatest effect on space debris levels as time goes on[1].
5. A private company that removes debris from space will make a set amount of revenue per piece of debris removed.
6. All pieces of space debris that de-orbit and enter Earth's atmosphere will burn upon entry. Although this cannot be guaranteed in reality, it is very rare that a piece of debris would actually reach the ground[2].
7. Reusable parts of a deployment system, given proper maintenance, will not become less reliable over time. This is justified because this is a minor detail that would greatly complicate the modeling of this problem.

3 Analysis of the Problem

3.1 Qualitative Analysis of Removal Systems

3.1.1 High-Powered Lasers

One proposed method for land-based removal of debris is to use high-powered lasers. In this method, lasers similar to those used for welding in factories are fired through telescopes at the debris. The small amount of momentum in the photons of the laser is used to nudge a piece of debris, slowing it down. If the piece of debris is small enough, the debris may even slow enough to de-orbit, enter the atmosphere, and burn upon entry. We did not consider this method feasible in this situation because the popular opinion in the scientific community is that these lasers do not carry enough momentum to deorbit debris larger than about 10 cm in diameter[11], and most of our targets are larger than that. Additionally, we will not consider space-based lasers because they have the same issue.

3.1.2 Gas Dispersion

Another proposed land-based method of debris removal is gas dispersion. In this method, focused pulses of atmospheric gases are dispersed into space, accelerating the decay of debris by creating a drag that slows and deorbits the debris. Once the debris deorbits, it enters the atmosphere and burns upon reentry. The pulses of gas fall back into the atmosphere shortly thereafter. We did not consider this method feasible because it cannot effectively target debris higher than 600 km[3], whereas low Earth orbit is anywhere between 160-2000 km[4], so a fair number of our targets may never be within reach of this method.

3.1.3 Electrodynamic Tethers

Electrodynamic tethers are one space-based option for removing space debris. In this removal method, a mother satellite is released into orbit. Once in orbit, a tether is released with a smaller child satellite attached to the end. Electricity flows through the tether, creating an electrodynamic drag on nearby debris, slowing them down considerably and de-orbiting the debris, leading to the debris entering the atmosphere and burning upon entry. The cost of this method is relatively unknown, but one advantage to this method is that it does not require a propellant. Instead, electrodynamic tethers generate thrust through interactions with Earth's magnetic field. Another benefit is that an electrodynamic tether can remove many debris in only one deployment. The main risk associated with this method is that the tethers are usually long, thin, and could be severed due to defects, vibrations, or contact with debris. However, most of these issues can be prevented with careful design and quality checks[16].

3.1.4 CleanSpace-One-Type Satellites

Perhaps the simplest method of removing space debris is using satellites similar to CleanSpace One. CleanSpace One is a proposed satellite that will be equipped with a folding conical net. After it is put into orbit, the satellite approaches its target and eventually surrounds it with its conical net. The net closes and the satellite gradually slows down until it de-orbits. Upon reentry, both the satellite and the debris burn up. Although it has not been fully tested, this method carries less risk than others because it does not rely on complex theories like other methods; its approach is simply to capture the debris. The cost of a satellite similar to CleanSpace One is not known, but it is not expected to be too expensive, at least relative to the cost of delivering it into space.

3.1.5 Deployment Methods

Rockets provide a tested but expensive method of reaching space. In most cases, the cheapest kind of rocket would be an unmanned, expendable rocket. A type of rocket that has been used for space debris missions in the past is the Japanese H-IIA rocket. The cost of an H-IIA rocket is estimated to be about $80 million[12]; however, a similar rocket currently in development – the H-III class rocket – is expected to be available very soon and has a cost of only $50-65 million[13]. For the most part, rockets have very little risk involved. The H-IIA rockets have a 96.5% success rate[14], and since they are expendable, a failure in any particular rocket launch would not be likely to end a debris removal program. Overall, rockets are the safest and most used method, and even though they are expensive, we will consider the use of rockets to be feasible in the given situation.

Another type of deployment system that we considered is a "skyhook". This method has been theorized since as early as 1966[5], and has undergone several iterations and research studies, the most recent of which is Boeing's HASTOL research program in 2001[7]. The proposed skyhook would be an orbiting, rotating structure comprised of a large counterweight with a long trailing cable attached to it such that the trailing end of the cable would reach down into the upper atmosphere every rotation. If the skyhook was deployed such that the orbital velocity and rotation rate were synchronized, the end of the cable would dip down to the atmosphere nearly vertically, following what is called a "cycloid curve"[6]. When the end of the cable is nearest the Earth, a hypersonic plane would travel to that point with the payload to be deployed, the grapple on the end of the cable would snag the payload, and pull it out of the plane. From there, the payload would be brought up to low Earth orbit by the rotation of the skyhook, where it would be released and the skyhook would continue its rotation for future deployments, while ascending to its previous orbit height to maintain the orbit. The deployment of a fully operational skyhook could theoretically lower the cost of deploying a payload to low Earth orbit from millions of dollars to only thousands of dollars. However, because it has never been done before, it would need to be researched, developed, built, and deployed, which would cost an estimated $18 billion. The risks regarding skyhook are hard to predict, as it hasn't been built yet, but the risks of this program would be inherently different from the risks of expendable rockets. There would be the catastrophic risk that the skyhook's initial deployment fails, which would end the program before money was returned on investment, as well as the risk that the skyhook would break after deployment and be very difficult or expensive to repair. Other than that, the biggest risk is that the skyhook fails to grab the payload or the plane misses the window of opportunity, in which case the payload is sent up on the next available cycle.

Although reusable rockets are no cheaper than expendable rockets, using a combination of usable and expendable technologies could create a cost-effective option. One notable deployment method is planned for use by a project in development called CleanSpace One. In this method, an A300 jetliner carries a reusable space shuttle, which is released after reaching an altitude of about 10 km. From there the space shuttle will fly to an altitude of about 80 km, where a rocket booster takes the satellite up a few hundred km and releases the satellite into orbit[15]. The cost of this method is much less than that of using rockets or a skyhook. Sources say that the cost of the CleanSpace One deployment will be about $10 million[15], but this includes the cost of building the CleanSpace One satellite, so the deployment itself probably costs even less. However, this method definitely has more risk than expendable rockets for several reasons: the system is very complex, it has not been tested, it has many stages, and a failure at any stage will result in a failed deployment. Overall, this could be the cheapest method of reaching space, but it also carries significant risk.

4 Parameters

Our model uses many parameters to determine the profit of different debris removal plans. To calculate total revenue, we use the revenue generated per piece of debris removed (See assumption 5 in section 2). To calculate the total cost, we use several kinds of sub-costs associated with each removal system. We consider a one-time upfront cost, which usually includes the planning of the mission and the construction of equipment, along with costs that occur for each new mission or deployment. We also include costs that occur over time, such as standard maintenance. Each of those sub-cost parameters is in turn modified using other parameters as well.

Since our model should be able to adapt to the plans of a private company, our model also uses parameters that the user may decide. Our model allows the user to specify the number of debris pieces that are to be removed from space per year of operation and the number of years over which the profit is to be maximized. Also, any of the theoretical data that we implemented can be replaced with more accurate numbers that a professional data team could find while planning of the project.

Each debris removal system has a certain level of risk associated with it. There is always a chance that an attempt at removing debris from space could result in failure. Our model accounts for this by taking in parameters that simulate different kinds of risk. This helps our model explore a variety of "What if?" scenarios and provide a better suggestions to the user.

5 Model Design

The problem asked us to determine "whether an economically attractive opportunity exists" for removing space debris, but every company will have different funding and constraints placed upon them. For this reason, we decided to make a model that would accommodate any company and their available options. We decided that as long as our model can provide the maximum profit attainable given any set of parameters, the company can then decide for themselves whether that price is "economically attractive". We also wanted a model that could elegantly test different combinations of debris removal systems. As such, we approached the problem as a linear programming problem.

5.1 Modeling a Debris Removal System

As stated, one of the goals of our model was to accommodate any options that are available to the company. To do this, we developed a general model for a debris removal system. We determined that the following pieces of information were sufficient to describe a generic system:

1. $cost_{static}$: Up-front cost of developing the system. This may include research and development, building of infrastructure or launch systems, or any other costs that only occur at the beginning of the program.
2. $cost_{dep}$: Cost per individual deployment of the system. For example, this might be the cost of a single rocket launch or skyhook delivery.
3. $cost_{annual}$: Annual cost of operation. This may include maintenance of systems, staff paychecks, or any other yearly costs.
4. $failcost$: Cost of standard failure for a single deployment. Such a failure would *not* be considered catastrophic, but still represents a failure to bring down debris. An example of this would be an expendable rocket exploding upon launch.
5. $failcost_{cat}$: Cost of catastrophic failure on a deployment. (See the paragraph below for a definition of catastrophic failure.)
6. $failcost_{cat_{all}}$: Cost of a catastrophic program failure (i.e. one which occurs before the system is ever deployed.)
7. $risk$: Risk of a regular deployment failure occurring
8. $risk_{cat}$: Risk of a catastrophic deployment failure occurring
9. $risk_{cat_{all}}$: Risk of a catastrophic program failure occurring
10. $\left(\frac{debris}{deployment}\right)$: Number of debris pieces deorbited in a single deployment
11. Dep_{max}: Maximum number of deployments per year

We define catastrophic failure as a failure of such great magnitude that the entire program must be shut down because of it. An example would be if a skyhook was destroyed; because the cost to build a skyhook is so great, if the skyhook was ever destroyed, the program would end immediately. Normal failure, on the other hand, is defined as a deployment that does not result in the removal of any debris, but does not destroy any expensive equipment that would be reused.

5.2 Modeling Profit of Debris Removal Systems

To quantitatively analyze the economic attractiveness of various removal systems, our model uses a linear program to maximize the profit of the company subject to a few constraints. The linear program in Equation 1 allows us to find the optimal combination of debris removal systems to maximize the company's profit, allowing them to determine for themselves if an economically attractive option exists. We will explain this linear program in detail, one component at a time.

$$
\begin{aligned}
\text{Maximize} \quad & \text{Profit} = \sum_{i=1}^{n} \text{Revenue}_i - \sum_{i=1}^{n} \text{Cost}_i \\
\text{subject to} \quad & \sum_{i=1}^{n} \text{Debris}_i \geq t \left(min \; \frac{debris}{year} \right) \\
& \sum_{i=1}^{n} \text{Debris}_i \leq t \left(max \; \frac{debris}{year} \right) \\
& Dep_i - Dep_{max_i} U_i \leq 0 \qquad\qquad \text{for } 1 \leq i \leq n \\
& Dep_i \geq 0, \; U_i \geq 0 \qquad\qquad\quad \text{for } 1 \leq i \leq n \\
& U_i \text{ is binary} \qquad\qquad\qquad\quad \text{for } 1 \leq i \leq n \\
\text{where} \quad & n \text{ is the number of debris removal systems} \\
& t \text{ is the number of years the company will operate} \\
& \text{Revenue}_i \text{ is the revenue generated by system } i \\
& \text{Cost}_i \text{ is the cost incurred by system } i \\
& \text{Debris}_i \text{ is the total debris collected by system } i
\end{aligned}
\tag{1}
$$

The decision variables in this linear program are Dep_i, the number of deployments per year of system i, and U_i, a binary variable indicating whether system i is being used or not.

We started by creating an objective function that represents the company's profit. This is calculated in the standard way, as the difference between revenue and cost. The revenue and cost are presented as sums because the total revenue and cost are the sums of the revenue and cost, respectively, from each of the n methods used.

Additionally, we formed a set of constraints to keep our results reasonable. The first two constraints restrict the minimum and maximum number of debris, respectively, that the company can collect overall. The left-hand side of each constraint sums up the total debris contribution (Debris_i) from each system to get the total number of debris collected over all t years. The parameters for minimum and maximum debris per year are represented in the model as $\left(min \; \frac{debris}{year} \right)$ and $\left(max \; \frac{debris}{year} \right)$. These two parameters are constant values that the company can choose based on external requirements and their own level of ambition. For example, government regulations may require the company to bring down at least five debris per year, but it is likely that the company's funding would put an upper limit on the number of debris they can collect.

The next set of constraints restricts the maximum number of deployments per year of each system. The maximum number of deployments of a satellite to space per year for system i is denoted as Dep_{max_i}. The construction of the constraints ensures that when $U_i = 0$, $Dep_i = 0$, and that when $U_i = 1$, $Dep_i \leq Dep_{max_i}$.

The final constraints simply restrict the decisions variables to be nonnegative, and restrict all the U_i variables to be binary.

5.2.1 Modeling Revenue Per System

$$\text{Revenue}_i = \left(\frac{revenue}{piece \; of \; debris} \right) \cdot \text{Debris}_i \tag{2}$$

Equation 2 shows the full formula for calculating revenue for a system. Using our assumption that the company would make a certain amount of revenue for each piece of space debris removed, the total revenue per system can be calculated as the revenue per debris times the total number of debris removed.

5.2.2 Modeling Debris Removed Per System

$$\text{DebrisActual}_i = \sum_{j=1}^{Dep_i \cdot t} \left(\left(1 - (risk_{cat_{all}})_i\right) \left(1 - risk_{cat_i}\right)^j \left(1 - risk_i\right) \left(\frac{debris}{deployment} \right)_i \right) \tag{3}$$

The total number of debris removed by a system can be written as the sum of the debris removed by each individual deployment. Equation 3 shows the full formula for calculating this. The upper limit of the sum, $Dep_i \cdot t$, is the total number of deployments of system i over all t years.

The number of debris pieces removed by any deployment is the product of the number of debris pieces removed by a single deployment and the probability that the deployment will be successful. The probability that the deployment will be successful depends on three risk factors defined in section 5.1: $risk_{cat_{all}}$, $risk_{cat}$, and $risk$. Since each risk parameter is a chance of failure, we subtract each parameter from 1 to get the chance of success.

The only term that requires further explanation is $(1 - risk_{cat_i})^j$. Since any catastrophic failure ends the program, this means that no more debris can be removed after a catastrophic failure. For each new deployment, then, the debris collected must account for the probability that a catastrophic failure has already occurred. To model this, we increase the exponent of $(1 - risk_{cat_i})$ by one for each additional deployment. This is because the probability that a catastrophic failure has not occurred by the end of the j^{th} deployment is $(1 - risk_{cat_i})^j$.

5.2.3 Modeling Cost Per System

$$\begin{aligned} \text{CostActual}_i = {} & U_i(cost_{static_i} + (failcost_{cat_{all}})_i(risk_{cat_{all}})_i + cost_{annual_i}t) \\ & + \sum_{j=1}^{Dep_i \cdot t} \big((cost_{dep_i} + failcost_i risk_i + failcost_{cat_i} risk_{cat_i})(1 - risk_{cat_i})^{j-1}\big) \end{aligned} \tag{4}$$

The cost of any particular debris removal system is calculated by combining many different costs and associated risks, as demonstrated in Equation 4. The U_i term deals with static costs and yearly operational costs. If $U_i = 1$, then the system is being used, and the company must pay these overhead costs. Otherwise, this system will incur no overhead costs for the company. The cost of a catastrophic system failure is "budgeted for" by multiplying $(failcost_{cat_{all}})_i$ by the risk of such a failure occurring.

The summation term deals with costs incurred on each deployment. The cost per deployment, failure cost, and catastrophic failure cost are added together to form the adjusted total cost per deployment. Both costs of failure are "budgeted for" similarly to the U_i term. Dep_i is constrained in such a way that if $U_i = 0$, $Dep_i = 0$, so the whole term will equal 0.

Since any catastrophic failure ends the program, no more costs can be incurred after a catastrophic failure. For each new deployment, then, the cost incurred must account for the probability that a catastrophic failure has already occurred. To model this, we increase the exponent of $(1 - risk_{cat_i})$ by one for each additional deployment.

5.3 Maintaining Linearity

One issue still remains: our DebrisActual_i and CostActual_i are not linear with respect to Dep_i. Both DebrisActual_i and CostActual_i, when simplified, contain a sum of the form $\sum_{j=1}^{Dep_i \cdot t}$, which places the decision variable Dep_i in the upper limit of the sum. When we use the formula for the partial sum of a geometric series to simplify these sums, we get a function which has Dep_i as an exponent to one of the terms. Equation 5 provides an example of this.

$$\sum_{j=1}^{Dep_i \cdot t} \big((1 - risk_{cat_i})^j\big) = (1 - risk_{cat_i}) \left(\frac{1 - (1 - risk_{cat_i})^{Dep_i \cdot t}}{risk_{cat_i}} \right) \tag{5}$$

In order to formulate our model as a linear program, we find linear approximations to these functions. Both DebrisActual_i and the second term of CostActual_i can be thought of as nonlinear functions of Dep_i. For example, DebrisActual_i takes in Dep_i and returns the total number of debris with Dep_i deployments. To find linear approximations of these functions, we examine the function values at their practical endpoints, when $Dep_i = 0$ and $Dep_i = Dep_{max_i}$. We then choose as our approximation the line passing through these two endpoints. Figures 1 and 2 show examples of what this line-fitting looks like.

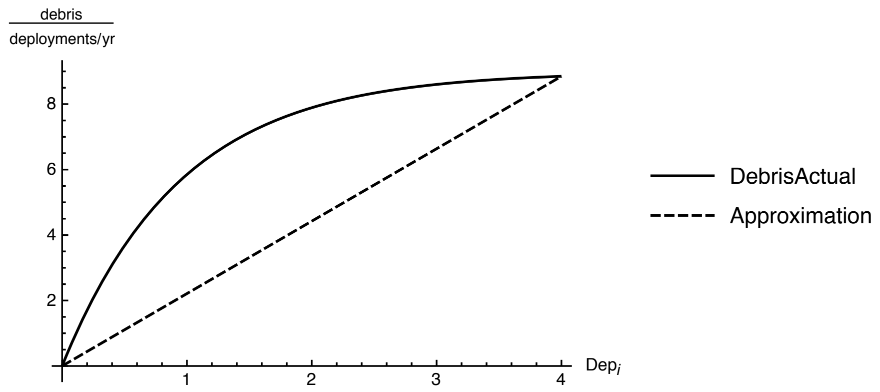

Figure 1: An exaggerated example of DebrisActual's curvature. In this figure, $risk_{cat} = 0.1$, a relatively high risk of failure.

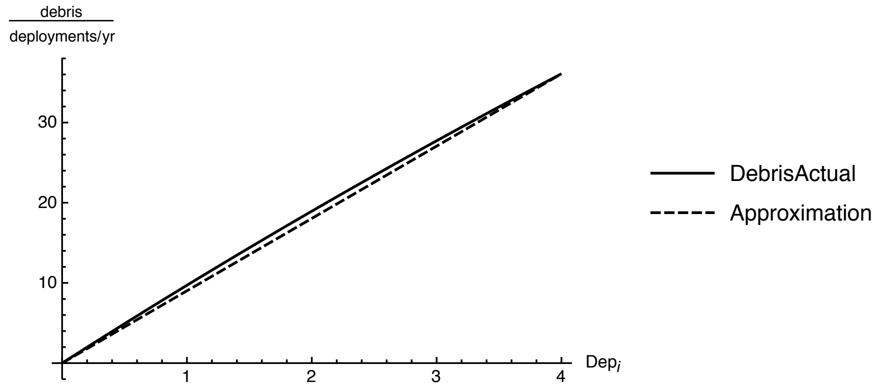

Figure 2: A more realistic example of fitting a line to DebrisActual, with $risk_{cat} = 0.005$.

Once we have this line, we use the line's slope as the coefficient to Dep_i. Using this process on both DebrisActual$_i$ and CostActual$_i$ results in the two functions in Equation 6, which can be safely used in a linear program.

$$\begin{aligned} \text{Debris}_i &= (slope_{debris})Dep_i \\ \text{Cost}_i &= U_i(cost_{static_i} + (failcost_{cat_{all}})_i(risk_{cat_{all}})_i + cost_{annual_i}t) + Dep_i(slope_{cost}) \end{aligned} \tag{6}$$

5.4 Solving the Linear Program

Since linearity has been established, we can proceed to solve the linear program. Once the parameters are all included, and the objective function and constraints are fully simplified, the model can be solved by any common linear program solver that supports mixed integer programming. In the results, the objective function value is the expected profit of the company at the end of t years. Additionally, any U_i with a value of 1 indicates a debris removal system which is in use, and each corresponding Dep_i indicates how many deployments per year are recommended to meet the company's goals.

For our own testing, we used a Java program to perform the calculations and generate LINDO code. We then used LINGO to solve the linear program and obtain detailed data about the results. This gave us the information we needed to provide our suggestions in section 6. However, our model can be implemented in any software with a linear program solver—even Microsoft Excel.

5.5 Possible Errors or Inaccuracies

Although our model provides accurate results that factor in many different costs, factors, and risks, a few components of our model could detract from its accuracy. One obvious place where some accuracy is lost is in our attempt to maintain the linearity of our program. As mentioned in section 5.3, we were forced to use a linear approximation of an exponential function instead of the original calculation. While the accuracy of this approximation diminishes when $risk_{cat}$ is large, we found the estimates to be very accurate when dealing with the very small estimates of risk we expect in the real world.

5.6 Collision Avoidance Sub-Model

Our model provides the user with the most economically attractive method of removing space debris as well as the expected amount of profit this method would yield, but we cannot say whether or not this method is economically attractive to any particular company. Because of this, we developed a supplementary sub-model that could provide an innovative alternative for avoiding collisions between orbiting space debris and objects in space that are currently in use.

One alternative for avoiding collisions is to predict where pieces of space debris will be at any particular time and check to see if their locations coexist at the same point of time. Since a private company may not have much data available to them about space debris, our sub-model takes in only minimal information about a piece of space debris that can be obtained from public records and determines its path parametrically.

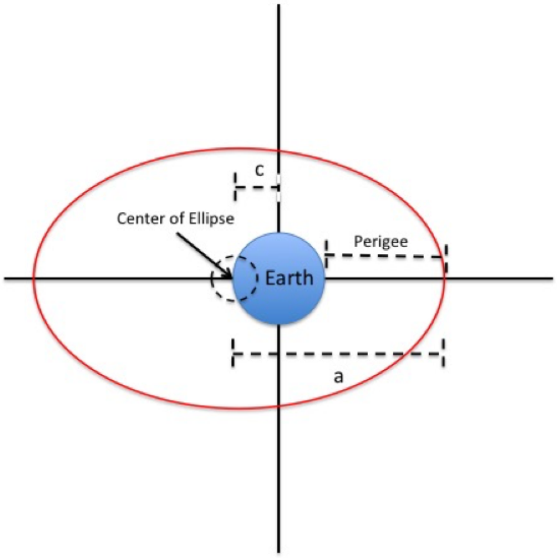

Figure 3: The value of c can be determined by the value of a, the perigee, and the radius of the Earth

Since orbits are elliptical with a large mass at one of the foci, we can assume that most debris have elliptical orbits with the Earth at a focus. The complete path of the piece of debris can be calculated with only the apogee and perigee of the debris, both of which are available to be found by the public. Since the apogee is the distance between the debris and the surface of the Earth when they are the furthest apart, and the perigee is the distance when they are closest, the sum of the apogee, perigee, and the diameter of the Earth is the length of the major axis of the ellipse, $2 * a$. Additionally, we can calculate the distance from the center of the ellipse to the foci, c, by finding the difference of b, the perigee, and the radius of the Earth, as shown in Figure 3. Once we find a and c a simple property of ellipses enables us to find b, as shown in

Equation 7.

$$c^2 = a^2 - b^2$$
$$b^2 = a^2 - c^2$$
$$b = \sqrt{a^2 - c^2} \tag{7}$$

Once we know a and b, we can write a parameterization of the path of the debris in the form $p(t) = (a * cos(t), b * sin(t), 0)$, assuming we project the path onto the xy plane.

To map the orbit onto the xy plane, we used the following approach:

1. Find two geographical location points in the satellite's orbit, preferably the most recent two
2. Because we assume that the center of the Earth is one focus of the elliptical orbit, plot those two points as vectors from the origin to provide two vectors in the plane of that satellite's orbit
3. Take the cross product of those two vectors to provide us with a vector that is orthogonal to the satellite's orbit
4. Take the cross product of the plane's orthogonal vector and the z-axis vector to provide an orthogonal vector about which the whole plane can be rotated while maintaining its form
5. Take the arccosine of the dot product between the plane's orthogonal vector and the z-axis to provide the angle by which the whole plane must be rotated to align its orthogonal vector with the z-axis
6. Use the generalized matrix for rotation about an arbitrary axis[9] to align the plane of the satellite's orbit with the xy plane

Once the elliptical path has been modeled simply in the xy plane, then the above steps are performed inversely such that the path gets mapped back to the satellite orbit's plane, resulting in a parametrically plottable elliptical orbit that depends on time as a parameter. This part of the collision avoidance model runs in linear time.

Once the parametric plot for each satellite and debris has been made, the next part of our plan is simple to state, but complicated to model: solve for t where satelliteA(t) = satelliteB(t) for all distinct pairs of satellites as $t \to \infty$. Unfortunately, we cannot think of a way to make this part of the collision avoidance model run in anything less than quadratic time due to its comparison of all distinct pairs of satellites. Because there are over 500,000 documented and tracked satellites and debris in orbit[8], and this runs in overall quadratic time, this method of collision avoidance would have a very hefty upfront computational cost. This upfront cost would come in exchange for knowledge of all future collisions for the current state of satellite orbits, as well as when each collision is about to occur. Assuming that the parametric plots are stored in a database of some kind, then for every course correction made to avoid one collision, the running time becomes linear again, as the new path would need to be checked against the other satellite paths, but the paths that don't collide would not need to be checked against each other again. At this stage, the collision avoidance could be done as far in advance as the company requires.

Future improvements to this method would be to implement some way to make sure that incoming satellite position data matches the corresponding parametric plot, and if it doesn't, the plot gets recalculated to be up-to-date. Something else that would be nice would be to update the parametric plot to generate its path given the 5 most recent positions of the satellite and the 5-point elliptical direct linear conversion[10]. The most obvious improvement would be to reduce the running time of this program as much as possible, due to the quadratic runtime bottleneck and the large amount of data that would need to be compared on the inital run. We prototyped this in Mathematica, but for this method to even be considered, we recommend writing the full program in a high-speed programming language like C.

6 Results

Our model took in a set of parameters and solved the resulting linear program to find the most profitable combination of systems. We tested the program with different parameters, varying the amount of revenue generated per debris removed, the duration of the program in years, and the minimum and maximum pieces of debris that could be removed from space.

First we tested our model using our best estimate of average parameters for a private company. This test was performed with the following parameters: the company would receive $5 million for each piece of debris removed, the company must remove at least 5 pieces of debris per year, and the company will not be paid for removing more than 18 pieces of debris per year. The results of that test are outlined in Figure 4 below.

As demonstrated by Figure 4, the initial costs of the program create a negative profit, if the program does not operate for at least six years. If the program runs for six years or longer, then it is expected to be profitable. After six years, the profit increases dramatically, making it highly advisable to plan for a long-term operation rather than a short-term one.

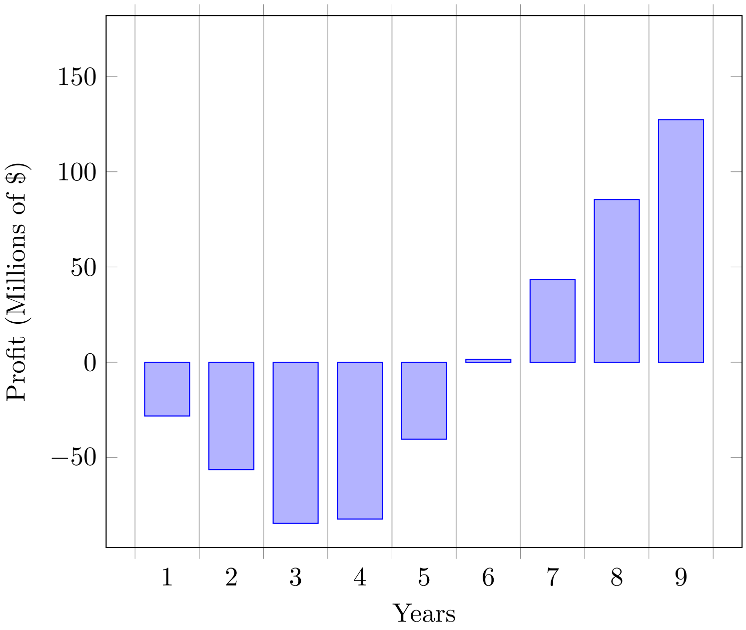

Figure 4: Net profit over multiple years at $5 million per debris.

6.1 Most Profitable Methods

One fact that is not demonstrated in Figure 4 is that the most profitable debris removal method changes depending on the number of years the program will operate. If the program will only operate for a year or so, the most profitable debris removal method is the use of rockets to bring satellites carrying electrodynamic tethers to space. However, if the program continues longer than that, as we suggest it should, the most profitable method would be to transport these satellites into space with the CleanSpace One transportation method.

Overall, the chosen parameters did not seem to matter too much in determining the most profitable method. The most profitable method was consistently the combination of the CleanSpace One transportation system and the electrodynamic tether debris removal system. Assuming that our data and choice of parameters are reasonable, the results of our testing suggest that this method of removing space debris would, in fact, be economically attractive to many companies.

7 Sensitivity Analysis

Because our model is largely dependent on parameters that could vary, our sensitivity analysis will be based on a situation with median values of parameters. Then we will then see how changing the parameters affects the optimal solution of the program. The parameters we analyzed are:

1. Years of program operation
2. Amount of revenue generated per debris removed
3. Minimum pieces of debris that the company must remove per year
4. Maximum pieces of debris that the client will pay for per year

For our sensitivity analysis, we had each of those parameters set to the following as the baseline we would compare against:

1. 10 years of operation
2. \$20 million per piece of debris
3. Minimum of 5 pieces per year
4. Maximum of 18 pieces per year

These parameters result in a profit of \$2.87 billion, using satellites with electrodynamic tethers deployed with the CleanSpace One method. Also, 18 pieces of debris were removed each year, the maximum that the company would be compensated for.

To analyze the model's sensitivity with respect to each of the listed parameters, we will change each parameter individually and compare the new results with our baseline.

7.1 Sensitivity With Respect to Years of Operation

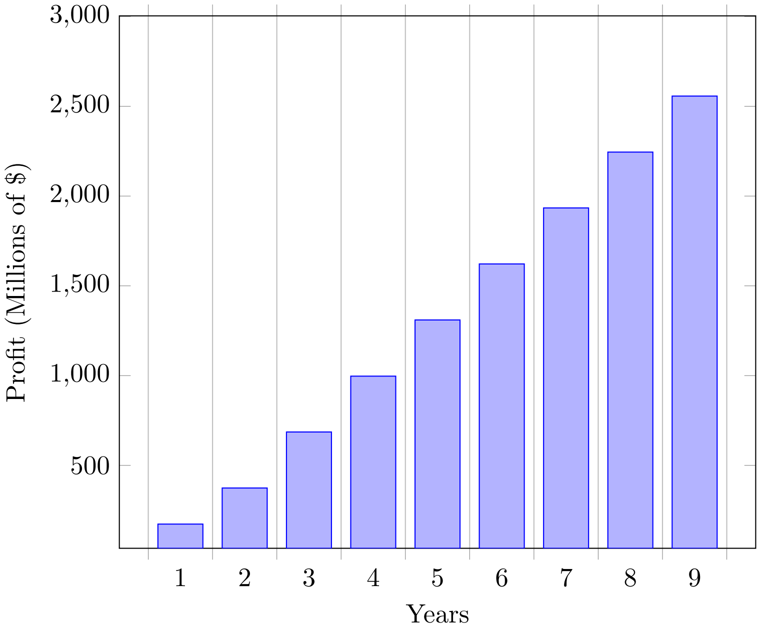

Figure 5: Net profit over multiple years at \$20 million per debris.

To thoroughly examine the change in the results over time, we tested the model with years of operation ranging from one to twenty in increments of one year. The expected profit increased linearly as the time was increased, ranging from \$174 million for one year to almost \$6 billion after twenty years, meaning that the years of operation parameter has a linear change on the profit at a rate of about \$291 million per year. The optimal method changed only once throughout that span; with one year of operation, rockets with satellites carrying electrodynamic tethers were the most profitable, but for two or more years of operation, the rockets were replaced with the CleanSpace One deployment system. In each case, 18 pieces of debris were removed, the maximum.

7.2 Sensitivity With Respect to Revenue Per Piece of Debris

When we change the revenue per piece of debris removed, the profit is directly affected and it has the possibility to change the number of pieces of debris removed. We tested the model with revenue per piece of debris removed varying from \$5-40 million in increments of \$5 million and examined the results with this parameter at \$1 million, also. The profit changed each time, as expected, increasing linearly with a rate of about \$180 million per \$1 million increase in the parameter. The only notable change is that, with a value of \$1 million, the optimal solution was to remove only 5 pieces of debris – the minimum – but for all other values of revenue per debris, it is optimal to remove 18 pieces – the maximum.

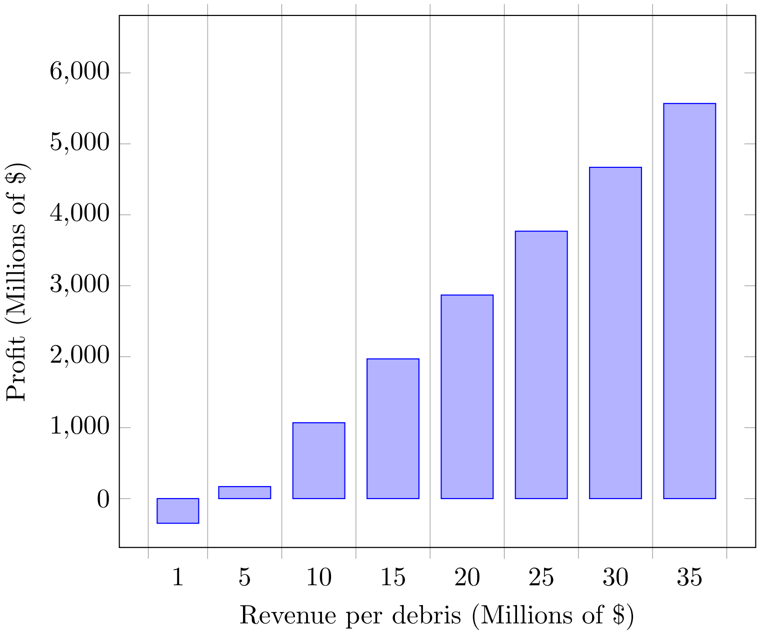

Figure 6: Net profit after 10 years with various revenue per debris.

7.3 Sensitivity With Respect to Minimum Pieces of Debris

We varied the minimum pieces of debris that the company must remove per year from 0-15 in increments of 5. As expected, this had no impact on the results, given that all other parameters were at their baseline value. This is because it is profitable to remove as much debris as possible with the current parameters. In each test, it was profitable to remove the maximum pieces of debris, so the minimum had no impact.

7.4 Sensitivity With Respect to Maximum Pieces of Debris

We tested the maximum pieces of debris per year for which the company will be compensated with values varying from 10-30 in increments of 5. In this range, the only significant change is seen in the profit. The same debris removal system is used in each case, and the maximum number of debris is always reached. The profit increases linearly as the maximum is increased, with values ranging from \$1.5 billion to almost \$5 billion, at a rate of about \$174 million per increased maximum debris per year.

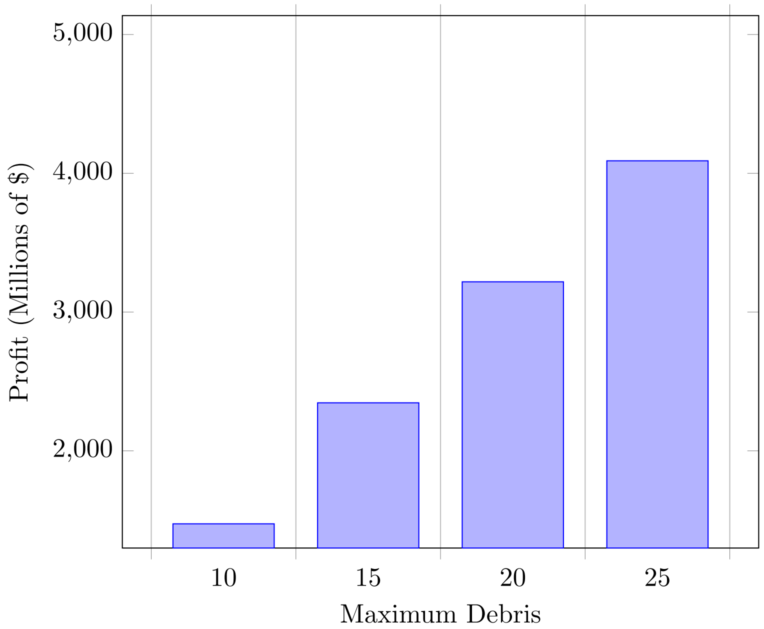

Figure 7: Net profit after 10 years with different maximum debris

8 Analysis of Our Model

8.1 Strengths

Our model is highly adaptable and can provide useful information to companies with different options that they are considering. As new and more accurate data becomes available, it can be implemented, and our model only becomes more accurate while others may become obsolete. Our model is also able to account for a multitude of different "What if?" scenarios because of its developed use of risk parameters. It is also able to account for this risk quantitatively and use it in our linear program. This model performs complex calculations to determine coefficients that create a simple linear program that can easily be solved. The solution provides ample information to the user including the expected profit and the optimal combination of alternatives for removing space debris. Additionally, if a company decides that the optimal debris removal method is not economically attractive, our sub-model provides an innovative alternative for avoiding collisions with space debris.

8.2 Weaknesses

While our model is quite robust, it has a few major weaknesses. One of them is that our model requires a number of parameters that are very difficult to obtain accurately and must be estimated or guessed, thus limiting the helpfulness of the results. Another weakness is that our model loses some accuracy at the points where a function must be linearly approximated. This occurs in two locations throughout the linear program. One final weakness is that a data team may have difficulty deciding which outcomes belong to the different kinds of risk. The classification of different kinds of risks are defined, but there is some subjectivity regarding which operational risk belongs in which category.

8.3 Future Improvements

Due to the limited time available to us, we had to make some simplifications that we would like to expand upon, given more time. We would like to develop a way to improve the accuracy of the model by improving the linear approximation used to maintain linearity. Perhaps some sort of best-fit line could be used instead of creating a line with the same endpoints as the original function. It would be ideal to maintain linearity, even if the nonlinear program could be solved, so it would be preferable to improve the approximation rather than remove it.

Another area we would like to improve is to model the time until a given system is ready to deploy. For example, a company could immediately use expendable rockets to take down debris, while researching and preparing potentially better solutions like CleanSpace One. At the moment our model makes no allowance for this.

We would also like to improve the determination of the values of the parameters. Given more time, we could do more thorough research and possibly model some of the parameters mathematically to obtain more accurate parameter values. However, this task is probably better suited for a professional data team and a group of aerospace engineers.

As a final point, we would also like to further develop our sub-model for collision avoidance. However, this falls slightly beyond the scope of this particular problem.

9 Conclusion

Our model gives specific profit predictions along with other information to let a company decide for themselves whether or not the most profitable alternative is economically attractive. However, based on the results of our testing, we would confirm that an economically attractive method for removing debris does, in fact, exist. This method would be to bring satellites carrying electrodynamic tethers into space with a method similar to the CleanSpace One method described in section 3.1.4. Our results suggest that over the course of a decade or so this method could accumulate a few billion dollars in profit, if implemented correctly.

In this paper we have described a robust and comprehensive model that can take in a set of parameters based on the preferences of any company and provide the most economically attractive method (or combination of methods) for removing space debris, along with the expected profit of using such a method. Our model can adapt and improve with more accurate data or more accurate parameter values, and it can account for a multitude of different scenarios using its risk parameters. Our model maintains simplicity by changing from a complicated, non-linear program into a clear and easy-to-read linear program. And in case the company does not find the most profitable option economically attractive, our sub-model can aid efforts to avoid collisions with space debris by predicting their paths.

We cannot decide whether or not an opportunity is economically attractive for any one company. There are simply too many unknowns and too many variables. But our model enables any company to decide for themselves whether or not collecting space debris is economically attractive.

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Executive Summary

In the search for the most economically attractive method to remove debris from space, a number of options and factors had to be considered. Initial research shows that the best strategy for solving the space junk problem is to remove large pieces of debris from space for the following reasons: they cause the most damage to other objects, they collide with other pieces of debris to create even more debris, and studies suggest that removing only a few large pieces of debris per year could have a significant impact. Another important fact is that the solution is not to destroy space debris. Destroying space debris just creates more small pieces of space debris, which are still dangerous and harder to remove than large pieces of debris. Since destruction is not an option, the best removal methods focus on slowing debris down. Slowing a piece of debris down will cause it to leave its orbit and start falling towards the Earth. As the debris enters the atmosphere, it heats up significantly, due to friction, and burns up before reaching the surface. With this in mind, we considered a number of different debris removal methods, including both land-based and space-based methods.

One proposed land-based method we analyzed was the use of high-powered lasers to slow pieces of debris and cause them to leave their orbits. Our analysis of this method led to the conclusion that this method would be relatively inexpensive and have a medium level of risk. The drawback to this method is that it most likely will not be effective on many of the larger pieces of debris that we will be targeting. Because of this, we do not suggest the use of land-based lasers. The other land-based method that we analyzed was gas dispersion. In this method, focused pulses of gas are released into space and create air drag, which slows the debris, causing it to leave its orbit. The cost of this method is relatively unknown at this point, but we expect it to be less than space-based methods. This method carries some risk because it has not been thoroughly tested on a large scale, so although the method should work in theory, we would be more confident in other methods. The other weakness of this method is that it is only effective for pieces of debris 600 km above the surface or lower. This limits its usefulness.

All space-based methods require both a method to reach space and a method to remove the debris. We examined rockets as one possible method of reaching space. Rockets have been tested more than any other method, so our data for this method is most reliable. Expendable rockets have a cost of about $50-80 million per deployment, but their risk is relatively low. We also analyzed a skyhook as a method of reaching space. Because skyhooks have not been tested on a large scale, we don't know how risky they could be. The largest downside to a skyhook is that it would cost billions of dollars to build. The advantage to using a skyhook is that once the skyhook is constructed, it can remove space debris at a much lower cost than rockets. A combination of vehicles can also be used to reach space. A promising method would be to carry a reusable rocket on top of a jetliner and release it about 10 km feet up. This has potential to be much cheaper than expendable rockets, but because there are more systems involved, there is more risk.

Based on our analysis, we would recommend that a private firm use rockets or a combination method to reach space because a skyhook would not be within the price range of a private firm, and it carries too much risk. One space-based debris removal system we analyzed was an electrodynamic tether. An electrodynamic tether is a structure consisting of a mother satellite and a child satellite that, when in use, are connected by a long, thin tether. Electricity flows through this tether, creating a drag that slows down debris and causes it to leave orbit. The cost of this method is relatively unknown but is expected to be inexpensive when compared to the cost of reaching space. This method has a moderate amount of risk, but an electrodynamic tether has been successfully released into space before, so this method is more proven than some others. We also looked into using satellites that have a folding net attached to them. Their purpose is to capture debris, gradually slow down, and enter the atmosphere, burning up on entry. This method is expected to cost about the same as the electrodynamic tether and have similar risk. This method has not been tested as much as the tether, but its straightforward system of removing the debris does not leave room for theoretical errors that could have occurred in the design of the tether. We also considered space-based lasers, but they had the same problems as the land-based lasers, so this method is not suggested.

After eliminating the infeasible options, we used our model to quantitatively analyze the economic attractiveness of various methods. The output of our model provides information including the expected profit and optimal method for space debris removal. After modeling each of the feasible combinations described earlier, we found that the optimal method of removing space debris is bringing satellites containing electrodynamic tethers into space with a combination of vehicles, as described earlier. Over a decade or so, if this plan is correctly implemented, we anticipate an accumulated profit in the billions. This accounts for the risk involved as well.

Based on our results, we would suggest that this plan is implemented. However, we cannot decide what is economically attractive for any particular individual, so we can provide any important information necessary to make that decision and leave it up to those in positions of authority.